

## Powers and Roots of Complex Numbers

### De Moivre's Theorem

If  $z = r(\cos \theta + i \sin \theta)$  is a complex number and  $n$  is any positive integer, then

$$z^n = r^n (\cos n\theta + i \sin n\theta)$$

### Examples:

a) Simplify  $(1+i)^6$ .

b) Simplify  $(\sqrt{3}-i)^4$ .

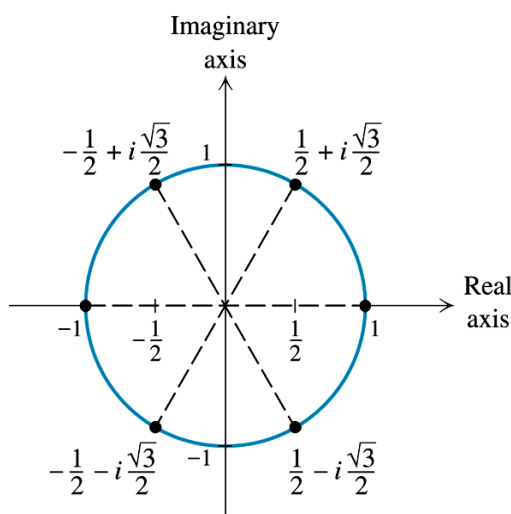
### Roots of a Complex Number

How many square roots does 4 have?

How many square roots does  $-9$  have?

How many sixth roots does 1 have?

It turns out that 1 has 6 sixth roots, and they are distributed evenly around the complex plane.



Sixth Roots of Unity

The complex number  $a+bi$  is an  $n$ th root of the complex number  $z$  if  $(a+bi)^n = z$ .

For any positive integer  $n$ , the complex number  $z = r(\cos \theta + i \sin \theta)$  has exactly  $n$  distinct  $n$ th roots given by

the expression  $r^{1/n} \left[ \cos \left( \frac{\theta + 360^\circ k}{n} \right) + i \sin \left( \frac{\theta + 360^\circ k}{n} \right) \right]$  for  $k = 0, 1, 2, \dots, n-1$ .

In radians, the roots are given by  $r^{1/n} \left[ \cos \left( \frac{\theta + 2k\pi}{n} \right) + i \sin \left( \frac{\theta + 2k\pi}{n} \right) \right]$  for  $k = 0, 1, 2, \dots, n-1$ .

The first of the  $n$  roots has an argument of  $\frac{\theta}{n}$ , and the other roots are spaced  $\left( \frac{360}{n} \right)^\circ$  apart.

(The circle is divided evenly into  $n$  pieces.)

**Examples:**

a) Find all of the fourth roots of the complex number  $-8-8i\sqrt{3}$ .

b) Find all the cube roots of 125.

c) Find all complex solutions to  $x^4 - 81 = 0$ .